

Magnetic flux detection with an Andreev Quantum Dot

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The charge of the subgap states in an Andreev quantum dot (AQD; this is a quantum dot inserted into a superconducting loop) is very sensitive to the magnetic flux threading the loop. We study the sensitivity of this device as a function of its parameters for the limit of a large superconducting gap Δ . In our analysis, we account for the effects of a weak Coulomb interaction within the dot. We discuss the suitability of this setup as a device detecting weak magnetic fields.

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Introduction. The Josephson effect [1] has been intensively studied during the past 45 years; its main characteristic is the presence of a tunable non-dissipative current when two bulk superconductors are joined via a normal or insulating layer and subjected to a superconducting phase difference φ . Recently, it has been realized that in a metallic junction the charge of the normal island in between the superconducting leads depends on the superconducting phase difference φ as well [2, 3]. This dependence is sufficiently strong [3] to use this effect in a magnetic flux detector, although our estimates below give a sensitivity somewhat below the sensitivity of the best SQUIDs.

Usually, small magnetic fields are measured by superconducting quantum interference devices (SQUIDs) [4, 5]. While SQUIDs are based on the dependence of the Josephson current on the superconducting phase difference φ (and hence on the magnetic flux Φ threading the loop), here we propose to use the charge-dependence in an Andreev quantum dot for the flux measurement. As shown in Ref. [3], the charge Q of a single-channel Andreev quantum dot can be fractional $-|e| < Q < |e|$ and depends on φ (here $e = -|e|$ is the charge of one electron).

The charge of an Andreev quantum dot can be measured by a sensitive charge detector, e.g., by a single-electron transistor (SET). Today, the best single electron transistors have a sensitivity of the order of $10^{-5} |e|/\sqrt{\text{Hz}}$ (e.g., see [6]). Using results of Ref. [3], simple estimates tell that an AQD can convert a change in flux $\delta\Phi$ to a change in charge δQ with a ratio $\delta Q/\delta\Phi \sim 2|e|/\Phi_0$, where $\Phi_0 = 2\pi\hbar/2|e|$ is the superconducting flux. Assuming a superconducting loop area $\sim 1 \text{ mm}^2$, we obtain the sensitivity $10^{-14} \text{ T}/\sqrt{\text{Hz}}$, which is comparable with the sensitivity $10^{-14} \div 10^{-15} \text{ T}/\sqrt{\text{Hz}}$

of today's best SQUIDs [4, 5]. Below, we study in detail the sensitivity ratio $\delta Q/\delta\Phi$.

Setup. Our Andreev quantum dot is realized by a small metallic dot connecting two superconducting banks joined in a loop, see Fig. 1. Our AQD is assumed to be a quasi one-dimensional normal metal (N) island separated from the superconductors (S) by thin insulator layers (I), generating normal scattering on top of the Andreev scattering characteristic of the normal-metal superconductor junction. The position of the normal resonance in this SINIS system can be tuned by the gate voltage V_g applied to the normal region of the AQD. The magnetic flux threading the loop Φ induces a superconducting phase drop φ across the AQD. Since

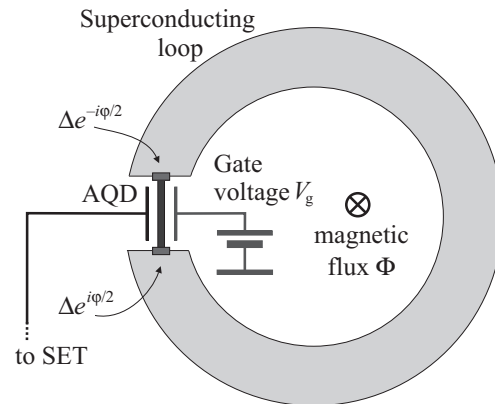


Fig. 1. Andreev quantum dot inserted into the superconducting loop. The Andreev quantum dot is connected to a single electron transistor (SET) and a gate electrode through capacitive couplings. The flux Φ produces a phase difference $\varphi = 2\pi\Phi/\Phi_0$ across the Andreev quantum dot. The charge of the AQD can be tuned by the gate voltage V_g and the flux Φ threading the loop.

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the phase drop in the bulk superconductor is negligible as compared to the phase drop φ across the AQD one may relate the latter to the flux Φ threading the loop, $\varphi = 2\pi\Phi/\Phi_0$. In order to measure the charge trapped on the AQD, a single electron transistor is capacitively coupled to the normal metal island. Experimentally, such AQDs have recently been fabricated by coupling carbon nanotubes to superconducting banks [7–10]. In the following, we concentrate on the properties of the key element in the setup — the Andreev quantum dot.

Energy and charge of the AQD without Coulomb interaction The Andreev states give rise to new opportunities for tunable Josephson devices, e.g., the Josephson transistor [11–13]; here, we are interested in their charge properties. We will consider the case of one transverse channel such that the problem effectively becomes one dimensional. We consider the case of a large separation δ_N between the resonances in the associated NININ problem (where the superconductors S have been replaced by normal metal leads N), $\delta_N \gg \Delta$, such that a single Andreev level ε_A is trapped within the gap region. We are interested in sufficiently well isolated dots with a small width Γ_N of the associated NININ resonance, $\Gamma_N \ll \Delta$. In this section, we neglect charging effects $E_C = 0$. In summary, our device operates with energy scales $\Gamma_N \ll \Delta \ll \delta_N$.

The resonances in the NININ setup derive from the eigenvalue problem $\hat{\mathcal{H}}_0\Psi = E\Psi$ with $\hat{\mathcal{H}}_0 = -\hbar^2\partial_x^2/2m + U(x) - \varepsilon_F$ with the potential $U(x) = U_{\text{ps},1}(x + L/2) + U_{\text{ps},2}(x - L/2) + eV_g\theta(L/2 - |x|)$ describing two point-scatterers²⁾ (with transmission and reflection amplitudes $T_l^{1/2}e^{i\chi_l^t}$, $R_l^{1/2}e^{i\chi_l^r}$; $R_l = 1 - T_l$, $l = 1, 2$) and the effect of the gate potential V_g , which we assume to be small as compared to the particle's energy E (measured from the band bottom in the leads), $eV_g \ll E$. Resonances then appear at energies $E_n = \varepsilon_L(n\pi - \chi_1^r/2 - \chi_2^r/2)^2$; they are separated by $\delta_N^n = (E_{n+1} - E_{n-1})/2 \approx 2E_n/n$ and are characterized by the width $\Gamma_N^n = T\delta_N^n/\pi\sqrt{R}$, where $\varepsilon_L = \hbar^2/2mL^2$. The bias V_g shifts the resonances by eV_g ; we denote the position of the n -th resonance relative to ε_F by $\varepsilon_N^n = E_n + eV_g - \varepsilon_F$. In the following, we choose a specific resonance in the gap by selecting an appropriate n and drop the index n , $\varepsilon_N^n \rightarrow \varepsilon_N$, $\delta_N^n \rightarrow \delta_N$, $\Gamma_N^n \rightarrow \Gamma_N$.

We go from a normal- to an Andreev dot by replacing the normal leads with superconducting ones. In order to include Andreev scattering in the SINIS setup, we have to solve the Bogoliubov-de Gennes equations (we choose states with $\varepsilon_A \geq 0$)

²⁾The Heaviside function $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x \leq 0$.

$$\begin{bmatrix} \hat{\mathcal{H}}_0(x) & \hat{\Delta}(x) \\ \hat{\Delta}^*(x) & -\hat{\mathcal{H}}_0(x) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \varepsilon_A \begin{bmatrix} u \\ v \end{bmatrix}, \quad (1)$$

with the pairing potential $\hat{\Delta}(x) = \Delta[\theta(-x - L/2)e^{-i\varphi/2} + \theta(x - L/2)e^{i\varphi/2}]$; $u(x)$ and $v(x)$ are the electron- and hole-like components of the wave function. The discrete states trapped below the gap derive from the quantization condition (in Andreev approximation)

$$(R_1 + R_2) \cos\left(2\pi\frac{\varepsilon_A}{\delta_N}\right) - 4\sqrt{R_1 R_2} \sin^2 \alpha \cos\left(2\pi\frac{\varepsilon_N}{\delta_N}\right) + T_1 T_2 \cos \varphi = \cos\left(2\alpha - 2\pi\frac{\varepsilon_A}{\delta_N}\right) + R_1 R_2 \cos\left(2\alpha + 2\pi\frac{\varepsilon_A}{\delta_N}\right).$$

The phase $\alpha = \arccos(\varepsilon_A/\Delta)$ is acquired at an ideal NS boundary due to Andreev reflection with $\varphi = 0$; the above formula can be directly obtained using results from Refs. [13, 14].

We concentrate on the regime $\Gamma_N, |\varepsilon_N| \ll \Delta$, the so-called $\Delta \rightarrow \infty$ limit. In this limit, the quantization condition can be expanded and we obtain the expression (A is the asymmetry parameter)

$$\varepsilon_A = \sqrt{\varepsilon_N^2 + \varepsilon_F^2}, \quad (2)$$

where

$$\varepsilon_F = \frac{\Gamma_N}{2} \sqrt{\cos^2 \frac{\varphi}{2} + A^2}, \quad A = \frac{|T_1 - T_2|}{2\sqrt{T_1 T_2}}. \quad (3)$$

The energy ε_A of the Andreev state is phase sensitive when ε_N is close to the chemical potential, $|\varepsilon_N| \lesssim \Gamma_N$, which can be achieved by tuning the gate potential V_g . In the limit $\Delta \rightarrow \infty$, both the $u(x)$ and $v(x)$ components of the wave function are nonzero only in the normal region,

$$\begin{bmatrix} u(x) \\ v(x) \end{bmatrix} = \begin{cases} 0, & |x| > L/2, \\ \begin{bmatrix} C_e^{\rightarrow} e^{ik_e x} + C_e^{\leftarrow} e^{-ik_e x} \\ C_h^{\leftarrow} e^{ik_h x} + C_h^{\rightarrow} e^{-ik_h x} \end{bmatrix}, & |x| < L/2, \end{cases}$$

where $k_{e,h} = [2m(\varepsilon_F \pm \varepsilon_A)]^{1/2}/\hbar$ are the wave vectors of electrons and holes, respectively. The coefficients are defined by $C_{e,h}^{\rightarrow} = C_{e,h}^{\leftarrow} = [(1 \pm \varepsilon_N/\varepsilon_A)/4L]^{1/2}$.

The ground state of the system is the state $|0\rangle$ with energy

$$\varepsilon_0 = \varepsilon_N - \varepsilon_A \quad (5)$$

(counted from the Fermi energy ε_F), where we have subtracted the energy of filled resonances below the Fermi surface; the latter are not modified by the superconductivity in the leads and hence do not depend on the phase φ . The first excited state with one Bogoliubov

quasiparticle is doubly degenerate in spin $|1_\uparrow\rangle = \hat{a}_\uparrow^\dagger|0\rangle$, $|1_\downarrow\rangle = \hat{a}_\downarrow^\dagger|0\rangle$ and has energy $\varepsilon_1 = \varepsilon_0 + \varepsilon_A = \varepsilon_N$. The doubly excited state with two quasiparticles $|2\rangle = \hat{a}_\uparrow^\dagger \hat{a}_\downarrow^\dagger|0\rangle$ has an energy $\varepsilon_2 = \varepsilon_0 + 2\varepsilon_A = \varepsilon_N + \varepsilon_A$.

The charge of the state $|\nu\rangle$ ($\nu = 0, 1_\uparrow, 1_\downarrow, 2$) can be obtained by differentiation of the corresponding energy ε_ν with respect to the gate voltage, $q_\nu = \partial\varepsilon_\nu/\partial V_g$, or by averaging the charge operator $\hat{Q} = e\sum_\sigma \int_{-L/2}^{L/2} \hat{\Psi}_\sigma^\dagger(x)\hat{\Psi}_\sigma(x)dx$ over the state $|\nu\rangle$, $q_\nu = \langle\nu|\hat{Q}|\nu\rangle$. Both methods give the identical results

$$q_0 = e\left(1 - \frac{\varepsilon_N}{\varepsilon_A}\right), \quad q_1 = e, \quad q_2 = e\left(1 + \frac{\varepsilon_N}{\varepsilon_A}\right). \quad (6)$$

Below, we will also need the off-diagonal matrix elements of the charge operator \hat{Q} ; the only non-vanishing term is $q_{02} = \langle 0|\hat{Q}|2\rangle = e(1 - \varepsilon_N^2/\varepsilon_A^2)^{1/2}$.

AQD with Coulomb interaction In order to find the effect of weak Coulomb interaction $E_C \ll \Delta$ in the limit Γ_N , $|\varepsilon_N| \ll \Delta$, we can disregard the continuous states with energies above the superconducting gap Δ and assume that the four levels of the discrete spectrum

form the entire basis of the system's Hilbert space³). The interaction is given by the operator

$$\hat{V} = E_C \frac{\hat{Q}^2}{e^2}. \quad (7)$$

Given the basis with these four states, we can diagonalize the Hamiltonian exactly. The non-zero matrix elements of the operator \hat{V} are

$$V_{00} = E_C(q_0^2 + q_{02}^2)/e^2, \quad V_{11} = E_C, \\ V_{22} = E_C(q_2^2 + q_{02}^2)/e^2, \quad V_{02} = 2E_C q_{02}/e. \quad (8)$$

The energy levels are defined by the eigenvalue problem

$$\begin{bmatrix} \tilde{\varepsilon}_0 - E & & & V_{02} \\ & \tilde{\varepsilon}_{1\uparrow} - E & & \\ & & \tilde{\varepsilon}_{1\downarrow} - E & \\ V_{20} & & & \tilde{\varepsilon}_2 - E \end{bmatrix} \begin{bmatrix} D_0 \\ D_{1\uparrow} \\ D_{1\downarrow} \\ D_2 \end{bmatrix} = 0,$$

where $\tilde{\varepsilon}_\nu = \varepsilon_\nu + V_{\nu\nu}$, $\nu = 0, 1_\uparrow, 1_\downarrow, 2$. The energy of the level with one Bogoliubov quasiparticle $|1\rangle$ is given by the (shifted) constant

$$E_1 = \varepsilon_N + E_C, \quad (9)$$

and does not mix with the other states; furthermore, the spin degeneracy of this Kramers doublet remains. The ground state $|0\rangle$ and the doubly excited state $|2\rangle$ mix due to Coulomb interaction and produce two new states, the singlet states $|-\rangle$ and $|+\rangle$; $|\pm\rangle = D_0^\pm|0\rangle + D_2^\pm|2\rangle$, $D_0^\pm/D_2^\pm = -V_{02}/(\tilde{\varepsilon}_0 - E_\pm)$, $|D_0^\pm|^2 + |D_2^\pm|^2 = 1$. The energies of these new states are

$$E_\pm = \varepsilon_N + 2E_C \pm \sqrt{(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2}. \quad (10)$$

The energies of the doublet and singlet states depend on ε_N and φ in a different way and may cross; thus the ground state can be formed by either the singlet $|-\rangle$ or by the doublet $|1\rangle$. The state $|+\rangle$ always remains the second excited state, see Fig. 2. When $E_C > \varepsilon_\Gamma \geq \Gamma_N A/2 \equiv E_C^*$ (with A the asymmetry parameter) the ground state is the doublet $|1\rangle$ in the region

$$-2E_C - \sqrt{E_C^2 - \varepsilon_\Gamma^2} < \varepsilon_N < -2E_C + \sqrt{E_C^2 - \varepsilon_\Gamma^2} \quad (11)$$

and remains $|-\rangle$ at all other values of ε_N [15].

The origin of this level crossing can be traced to the different shifts in energies with E_C : While E_1 is shifted up by E_C , E_- quickly approaches 0 with increasing E_C .

³In realistic nanodevices the Coulomb energy can be larger than Γ_N and smaller or of the order of δ_N , but in principle can be made much smaller than both δ_N and Δ (see the discussion in [3, 7]).

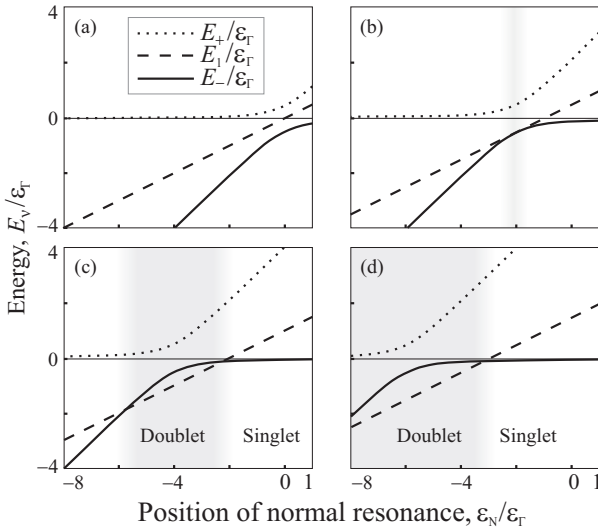


Fig. 2. Energies E_- (solid line), E_1 (dashed line), and E_+ (dotted line) are plotted versus the position of normal resonance. All energies are given in units of ε_Γ , cf. (3). The Coulomb energy is $E_C = 0$ for (a), $E_C = \varepsilon_\Gamma$ for (b), $E_C = 2\varepsilon_\Gamma$ for (c), and $E_C = 3\varepsilon_\Gamma$ for (d). In accordance with formula (11) the doublet region appears when $E_C > \varepsilon_\Gamma$, see (b–d). In the filled region the ground state of the system is a doublet; the width of this region is $2(E_C^2 - \varepsilon_\Gamma^2)^{1/2}$, the edges of this region are spread due to the finite temperature Θ .

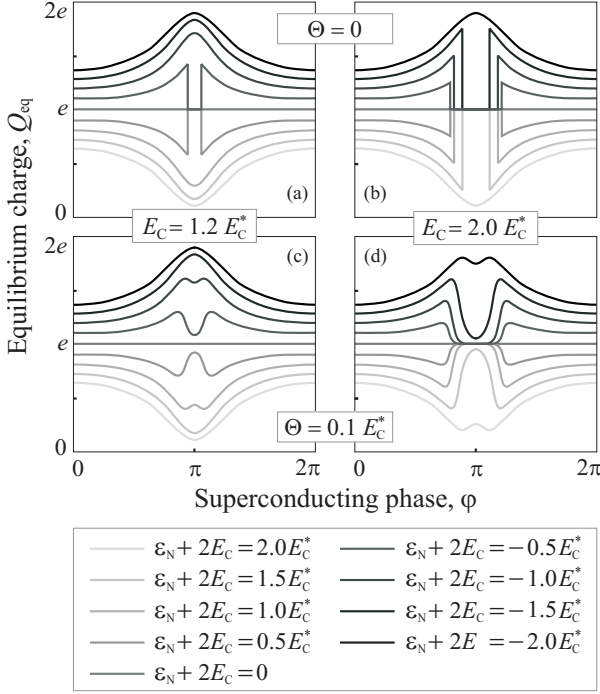


Fig. 3. Equilibrium charge Q_{eq} (15) versus superconducting phase difference φ . In (a) and (b) the temperature is zero (i.e., Q_{eq} represents ground state charge), in (c) and (d) the temperature is $\Theta = 0.1 E_C^*$, where $E_C^* \equiv \Gamma_N A/2$. The Coulomb energy is $E_C = 1.2 E_C^*$ for (a) and (c), $E_C = 2.0 E_C^*$ for (b) and (d). The asymmetry level of the dot is $A = 0.2$. The features in the center of the plots corresponds to the Kramers doublet region (11). In (c) and (d) the border of the doublet region is smoothed by the temperature Θ .

Note that the terms $\propto q_{02}^2$ and $\propto q_{02}$ in the matrix elements $V_{\nu\nu'}$ lead to the crossing of the energies E_- and E_1 , while preventing the crossing of the level E_+ with the others.

At the edge of the region (11) a sharp singlet to doublet crossover takes place, with a jump appearing as a function of $\varepsilon_N(V_g)$ or $\varepsilon_\Gamma(\varphi)$ in the charge of the Andreev dot and in the current across (see below). The charges of the new states $|\mu\rangle$, ($\mu = 1, \pm$) can be calculated as in the previous section, $Q_\mu = \partial E_\mu / \partial V_g$, and are given by

$$Q_\pm = e \left(1 \pm \frac{\varepsilon_N + 2E_C}{\sqrt{(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2}} \right), \quad Q_1 = e. \quad (12)$$

The charge Q_1 is integer and does not fluctuate; the charges Q_\pm are fractional in the region $|\varepsilon_N + 2E_C| \sim \varepsilon_\Gamma$ and fluctuate strongly (see also the discussion of fluc-

tuations in Ref. [3] where Coulomb effects have been ignored)

$$\delta Q_\pm \equiv [\langle \pm \hat{Q}^2 | \pm \rangle - \langle \pm \hat{Q} | \pm \rangle^2]^{1/2} = e \frac{\varepsilon_\Gamma}{\sqrt{(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2}}. \quad (13)$$

Note that the Coulomb interaction merely shifts the regime of ε_N where the charges Q_\pm are fractional. Everywhere outside the doublet region the ground state charge is given by Q_- , while within the Kramers doublet region the charge is pinned to the value $Q_1 = e$. As illustrated in Figs. 3a and 3b, for $E_C > E_C^*$ a sharp crossover occurs and the charge jumps by the value $\delta Q_{\text{cr}} = Q_- - Q_1$. This jump is smeared at finite temperatures, see Figs. 3c and 3d. The groundstate charge is

$$Q_{\text{gs}} = e - e \frac{E_N}{E_A} \theta[E_C < E_A],$$

where $E_A = [(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2]^{1/2}$ and $E_N = \varepsilon_N + 2E_C$ denotes the energy of the shifted normal state resonance. The equilibrium charge at finite temperature Θ is

$$Q_{\text{eq}} = \frac{Q_- e^{-E_-/\Theta} + 2Q_1 e^{-E_1/\Theta} + Q_+ e^{-E_+/\Theta}}{e^{-E_-/\Theta} + 2e^{-E_1/\Theta} + e^{-E_+/\Theta}}; \quad (15)$$

here and below we set Boltzmann's constant $k_B = 1$. The equilibrium charge as a function of the superconducting phase $\varphi = 2\pi\Phi/\Phi_0$ is shown in Fig. 3.

The currents in the states $|\mu\rangle$ are defined by relationship $J_\mu = \partial E_\mu / \partial \Phi$ which provides the results

$$J_\pm = \mp \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2 \sin \varphi}{16 E_A}, \quad J_1 = 0. \quad (16)$$

The groundstate current is

$$J_{\text{gs}} = \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2 \sin \varphi}{16 E_A} \theta[E_C < E_A];$$

note that the current vanishes throughout the doublet region. The thermal equilibrium current is

$$J_{\text{eq}} = \frac{J_- e^{-E_-/\Theta} + J_+ e^{-E_+/\Theta}}{e^{-E_-/\Theta} + 2e^{-E_1/\Theta} + e^{-E_+/\Theta}}. \quad (18)$$

Differential sensitivity The differential sensitivity of the equilibrium charge to the magnetic flux threading the superconducting loop is defined by the absolute value of the derivative $\partial Q_{\text{eq}} / \partial \Phi$ taken at the given value of flux,⁴ $S = |\partial Q_{\text{eq}} / \partial \Phi|$. By using (15) we obtain

$$S = \left| F_\Theta \frac{\partial Q}{\partial \Phi} + Q \frac{\partial F_\Theta}{\partial \Phi} \right|, \quad (19)$$

⁴Note that the sensitivity of the charge-to-flux convertor $S \equiv S_{\Phi \rightarrow Q}$ coincides with the voltage-to-current sensitivity of the Josephson transistor described in Ref. [13] $S_{V \rightarrow J} = |\partial J_{\text{eq}} / \partial V_g|$.

where $Q \equiv (Q_+ - Q_-)/2$, the derivative

$$\frac{\partial Q}{\partial \Phi} = e \frac{2\pi}{\Phi_0} \frac{E_N \Gamma_N^2 \sin \varphi}{16E_A^3}, \quad (20)$$

the function

$$F_\Theta = \frac{e^{-E_+/\Theta} - e^{-E_-/\Theta}}{e^{-E_-/\Theta} + 2e^{-E_1/\Theta} + e^{-E_+/\Theta}} = -\frac{\sinh(E_A/\Theta)}{\cosh(E_A/\Theta) + e^{E_C/\Theta}}, \quad (21)$$

and its derivative

$$\frac{\partial F_\Theta}{\partial \Phi} = \frac{e^{E_C/\Theta} \sinh(E_A/\Theta) + 1}{[\cosh(E_A/\Theta) + e^{E_C/\Theta}]^2} J_-. \quad (22)$$

As illustrated in Fig. 3 there are two intervals where the $Q_{\text{eq}}(\varphi)$ dependence is steep. As φ increases from $\varphi = 0$, the charge increases (decreases) and reaches a maximum (minimum). For $E_C < E_C^*$ the maximum (minimum) of the charge is always at $\varphi = \pi$, while for $E_C > E_C^*$ the extremum splits and a second interval with a steep dependence $Q_{\text{eq}}(\varphi)$ emerges in between the two new extrema. The first interval (interval I in what follows) corresponds to the singlet state of the AQD, the second (interval II in what follows) corresponds to the doublet state. We start with a description of the first interval. We fix the parameters Γ_N , A , and E_C and search for the maximum sensitivity S as a function of ε_N and φ . The non-trivial symmetries $Q_{\text{eq}}(\varphi, \varepsilon_N) = Q_{\text{eq}}(2\pi - \varphi, \varepsilon_N)$, $Q_{\text{eq}}(\varphi, \varepsilon_N) - Q_{\text{eq}}(\varphi, 0) = -Q_{\text{eq}}(\varphi, -\varepsilon_N - 4E_C) + Q_{\text{eq}}(\varphi, 0)$ allow us to restrict the search to the region $0 \leq \varphi \leq \pi$, $\varepsilon_N + 2E_C \geq 0$. Subsequently, we analyze the maximum as a function of E_C keeping A and Γ_N constant.

Interval I: For $E_C < [3(1+A^2)/(1+2A^2)]^{1/2} E_C^*$ and zero temperature $\Theta = 0$ the sensitivity is determined by the derivative $\partial Q/\partial \Phi$ (20). The function $|\partial Q/\partial \Phi|$ has a maximum at $\varepsilon_N + 2E_C = [(1+A^2)/(1+2A^2)]^{1/2} E_C^*$ and $\varphi = \pi - 2 \arcsin[A/(1+2A^2)^{1/2}]$, where the differential sensitivity is given by

$$S_{\text{max}}^{\text{I}} = |e| \frac{2\pi}{\Phi_0} \frac{1}{6\sqrt{3}A\sqrt{1+A^2}}. \quad (23)$$

One observes that the smaller A is, the larger is the sensitivity. In other words, a symmetric SINIS structure provides a better sensitivity $S_{\text{max}}^{\text{I}}(A \rightarrow 0) \rightarrow \infty$, but at the same time the region in φ with this large sensitivity vanishes as $A \rightarrow 0$. When $\Theta \ll E_C^*$ the sensitivity is nearly independent of temperature.

In the opposite case $E_C \geq [3(1+A^2)/(1+2A^2)]^{1/2} E_C^*$ the doublet region covers all of the interval I and the maximum at zero temperature is always realized at the

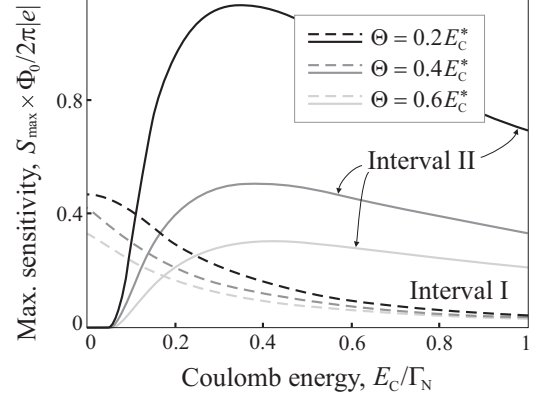


Fig. 4. Maximum of the differential sensitivity S_{max} (absolute value) in the interval I (dashed lines) and in the interval II (solid lines) versus Coulomb energy E_C at the asymmetry level $A = 0.2$, $E_C^*/\Gamma_N = 0.1$. The temperature varies from $\Theta = 0.2E_C^*$ up to $\Theta = 0.6E_C^*$.

edge of the doublet region (11), with a sensitivity given by

$$S_{\text{max}}^{\text{I}} = |e| \frac{2\pi}{\Phi_0} \frac{\Gamma_N^3}{48\sqrt{3}E_C^3} \times \sqrt{2(\lambda^2 - \lambda + 1)^{3/2} - (\lambda + 1)(\lambda - 2)(2\lambda - 1)} \quad (24)$$

realized at $\varepsilon_N + 2E_C = (\Gamma_N/2)\{[2\lambda - 1 + (\lambda^2 - \lambda + 1)^{1/2}]/3\}^{1/2}$ and $\varphi = 2 \arccos\{[\lambda + 1 - (\lambda^2 - \lambda + 1)^{1/2}]/3\}^{1/2}$, where $\lambda = (E_C^2 - E_C^{*2})/(\Gamma_N/2)^2$. This result reduces to

$$S_{\text{max}}^{\text{I}} \approx |e| \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2}{16E_C^2} \quad (25)$$

in the limit $E_C \gg \Gamma_N$, and remains approximately correct for $E_C \approx \Gamma_N/2$. For $E_C \gg \Gamma_N$, the maximum sensitivity is reached at $\varepsilon_N + 2E_C \approx E_C - \Gamma_N^2/16E_C$ and $\varphi \approx \pi/2 + \Gamma_N^2/16E_C^2$.

Interval II: At zero temperature there is a jump in the charge at the edges of interval II and thus the sensitivity diverges in these points. A finite temperature smears the jump and the sensitivity becomes finite. If $E_C \gg \Theta$, Γ_N , E_C^* , the sensitivity S reaches the maximum near the point $\varepsilon_N + 2E_C = E_C$, $\varphi = \pi/2$ where it equals to

$$S_{\text{max}}^{\text{II}} \approx |e| \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2}{64E_C\Theta}. \quad (26)$$

The expression for $S_{\text{max}}^{\text{II}}$ is too cumbersome for an arbitrary Coulomb energy E_C and we plot the numerical result $S_{\text{max}}^{\text{II}}(E_C)$ in Fig. 4. In the same plot, we also present the maxima of the sensitivity from the interval I. One easily notes that for a large Coulomb interaction the charge jump smeared by temperature provides the sharper $Q_{\text{eq}}(\varphi)$ dependence.

Conclusion. In this article, we have pointed out that the φ -dependence of the charge trapped within an Andreev quantum dot may be used for the implementation of a new type of magnetometer which operates along the pathway ‘magnetic flux–AQD charge–SET–current’ instead of the usual direct SQUID scheme ‘magnetic flux–current’. We have analyzed the charge sensitivity as a function of magnetic flux, gate voltage, Coulomb interaction, dot asymmetry, and temperature. The sensitivity of our setup can be further increased by adding an electromechanical element [16]: Applying a large electric field to the charged nanowire, the change in charge will lead to a mechanical shift of the wire. This shift can then be detected due to the change in the capacitance of the compound setup as in Ref [16]. In the present work, we have concentrated on a single-channel wire in order to demonstrate the effect; the case of an n -channel wire ($n = 2$ or $n > 2$) can be analyzed using the same technique and we plan to study this case in the near future.

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